Strings, Fields and Holographic Correspondence

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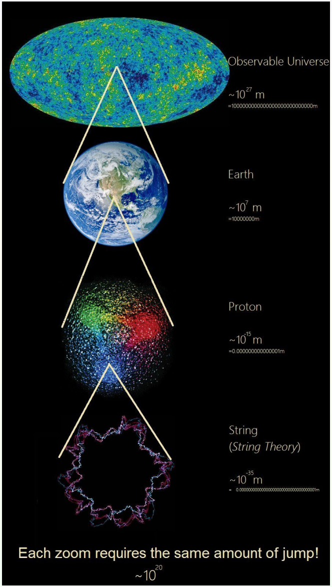
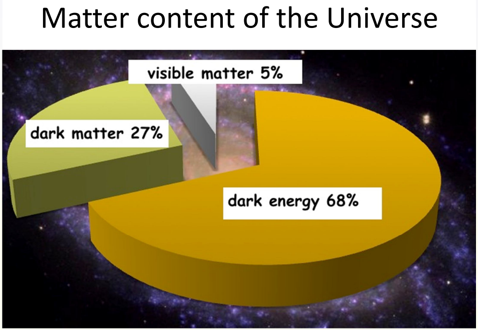
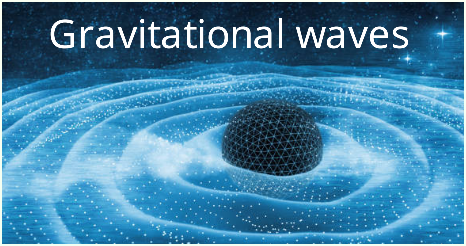
Fundamental problems and why holographic correspondence?







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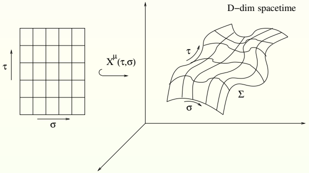

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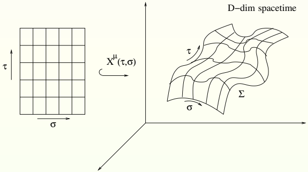


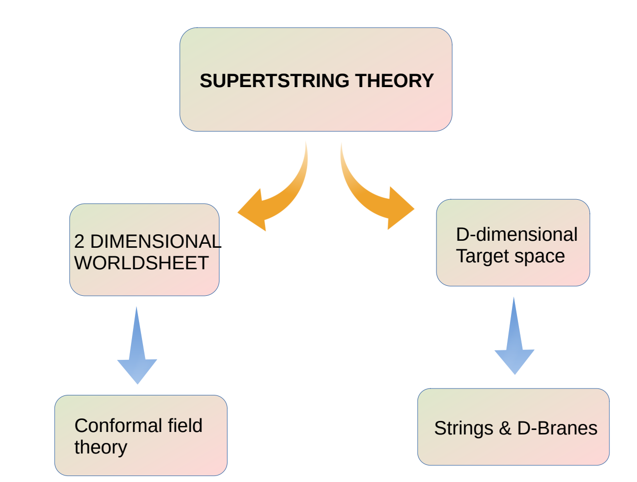
• Two aspects of (super)string theory:

Two theories within String theory

Dynamics of Strings : $\begin{cases} \text{- Quantum 2d worldsheet CFT} \\ \text{- Target space dynamics of worldsheets} \end{cases}$

String worldsheet

- Conceptual issues:



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- Φ-coupling

$$S_\Phi = -\frac{1}{4\pi}\int d^2\sigma\sqrt{g}\,\Phi(X^\mu)R^{(2)}.$$

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• The energy-momentum tensor is:

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}\eta_{\alpha\beta}\partial^\rho X^\sigma \partial_\rho X_\sigma = 0.$$

• The conservation conditions

$$\partial^\alpha T_{\alpha\beta} = 0, \quad , T^\alpha{}_\alpha = 0.$$

Define $\Theta(z) = T^{00} + T^{01}$ and $\bar{\Theta}(\bar{z}) = T^{00} - T^{01}$. The algebra closed by $\Theta(z)$ is the so-called $\boxed{\text{Virasoro algebra}}$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}, \quad \Theta(z) = \sum_n \frac{L_n}{z^{n+2}}.$$

• Requiring above symmetries of the σ-model string action, one ends up with the conditions ensuring the vanishing of the corresponding β-functions:

$$\beta^G_{\mu\nu}: \quad R_{\mu\nu} - \frac{1}{4} H_{\mu\sigma\lambda} H_\nu^{\sigma\lambda} + 3 D_\mu \partial_\nu \Phi = 0,$$

$$\beta^B_{\mu\nu}: \quad -\frac{1}{2} D^\sigma H_{\sigma\mu\nu} + H_{\sigma\mu\nu} D^\sigma \Phi = 0,$$

$$\beta^\Phi: \quad \frac{1}{6}\left[d - 10\right] - \frac{\alpha'}{2}\left[D^2\Phi - 2(\nabla\Phi)^2 - \frac{1}{12}H^2\right] = 0.$$

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\end{aligned}$$

$\boxed{\text{The effective 10d action from closed strings}}$

$$S = \frac{1}{2\kappa}\int d^{10}X \sqrt{|G|} e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{12}H^2\right)$$

$\surd$ should be understood as a universal one, i.e. any superstring background must satisfy the above equations

$\surd$ the equations of motion following from this action coincide with the conditions ensuring vanishing of the β-functions.

Open strings

- adding open string sector
  In the case of boundaries of the world sheet one can write the action as

$$S = \frac{1}{1\pi\alpha'}\left(\int_\Sigma d^2\sigma \frac{1}{2}(\partial_\alpha X_\mu \partial^\alpha X^\mu \right.$$
$$\left. + \varepsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu) + \int_{\partial\Sigma} d\sigma^1 A_\mu \partial_1 X^\mu\right)$$

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- boundary conditions:

$$(\mathbf{NN}):\ \partial_\sigma X^\mu_{|\sigma=0,\pi} = 0 \implies$$
$$X^\mu(\tau,\sigma) = q^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n \cos n\sigma e^{-in\tau}$$

$$(\mathbf{DD}):\; X^\mu_{|\sigma=0} = q_i^\mu,\; X^\mu_{|\sigma=\pi} = q_f^\mu \implies$$

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- expanding the above action we get model dependent terms

$$S^{II}_{open} = -\frac{1}{2\kappa^2}\int d^{10}x \sum_p \frac{1}{2(p+2)!}F^2_{p+2},$$

where F_{p+2} is the field strength of a $p+1$ form gauge field. The couplings A_μ get promoted to gauge fields on the subspace where string endpoints live.

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- Stack of N parallel D branes

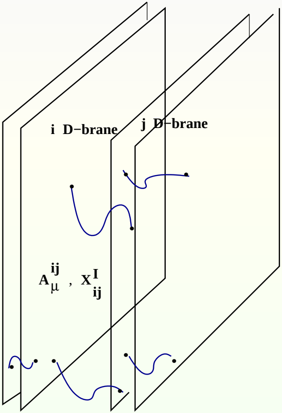


Figure: The degrees of freedom N parallel D-branes.

Effective open string action sources

type		form	el. source	mag. source
IIA	10d N=2 SUSY non-chiral	$F_{[2]}$	D0	D6
IIA	10d N=2 SUSY non-chiral	$F_{[4]}$	D2	D4
IIB	10d N=2 SUSY chiral	$F_{[1]}$	D(-1)	D7
IIB	10d N=2 SUSY chiral	$F_{[3]}$	D1	D5
IIB	10d N=2 SUSY chiral	$F_{[5]}$	D3	D3

Table: Field strengths and sources in type II strings

IIA: two gravitini moving in opposite directions on the closed string world sheet, opposite chiralities under the 10d Lorentz group: a non-chiral theory.
IIB: the gravitini have the same chiralities under 10d Lorentz group: chiral theory
$\rightarrow$ The main conclusion one can draw is that the $(p+1)$-dimensional world-volume serves as source of $(p+2)$-form field $F_{[p+2]}$.

Gauge theories on Dp brane

$$S_{Dp} = S_{DBI} + S_{WZ}\,,$$
$$S_{DBI} = -T_p \int_{Dp} d^{p+1}\xi\, \mathsf{STr}\sqrt{-\det\left(\mathcal{P}_{ab}[G_{\mu\nu} + B_{\mu\nu}] + 2\pi\alpha' F_{ab}\right)}\,,$$
$$S_{WZ} = T_p \int_{Dp} \sum_i \mathsf{STr}\,\mathcal{P}[C_{(i)}] \wedge e^{\mathcal{P}[B]+2\pi\alpha' F}\,,$$

where T_p is the brane tension.

$$T_p = \frac{1}{g_S\,(2\pi)^p \alpha'^{(p+1)/2}}\,.$$

The pull-back of the background metric $G_{\mu\nu}$ and Kalb-Ramond-field $B_{\mu\nu}$ is denoted by $\mathcal{P}$. STr is the symmetrized trace.

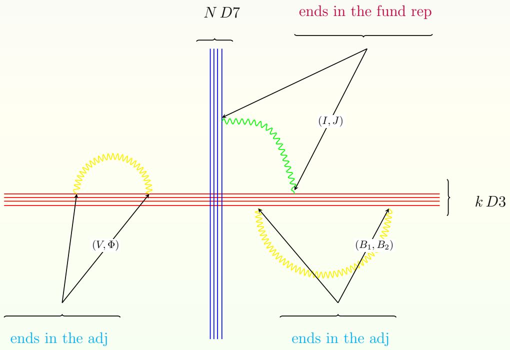
- Expand the brane action and obtain the leading contribution

$$S = T_p \int d^{p+1}\xi \sqrt{g} e^{-\phi} (2\pi\alpha')^2 \frac{1}{2} \mathrm{tr}\left(F_{\alpha\beta}F^{\alpha\beta}\right) + T_p \int \sum_r C^{(r)} \wedge \mathrm{tr} e^{2\pi\alpha' F} + \cdots , \quad (1)$$

where we have not written terms involving fermions and scalars. This action is the sum of

- a Yang-Mills term,
- a Wess-Zumino term,
- an infinite number of corrections at higher orders in α' indicated by $\cdots$ in (1)

- The string endpoints on the same D_p branes transform under adjoint representation of the gauge group. The large number of these branes gives the background geometry.
- The string endpoints ending on different $D_p - D_{p'}$ branes transform in the fundamental. This is the way we introduce flavors in the theory.



Example: D3-brane

The solution for the $D3$ branes can be obtained from the above general solutions.

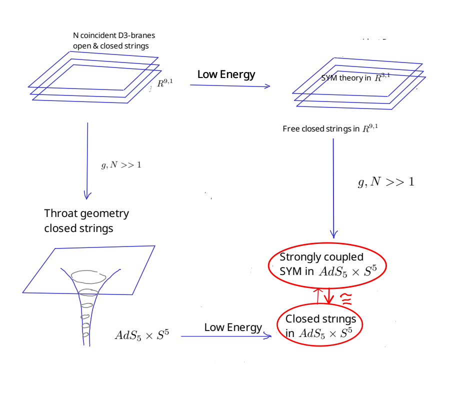
$$S = \frac{1}{2\kappa^2} \int\limits_M d^{10} \mathcal{L}_{(10)} + \int\limits_\Sigma d^4 z \mathcal{L}_{D3}, \tag{2}$$

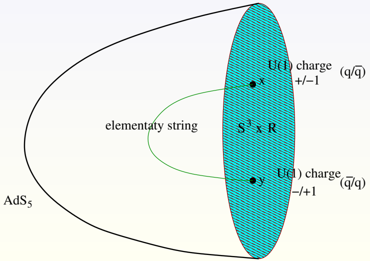
where we omitted possible interactions.
The explicit form of the $D3$-brane solution is

$$D3\text{-brane} = \begin{cases} ds^2 = H^{-\frac{1}{2}} dx^2_{(4)} + H^{\frac{1}{2}} \left(dy^2 + y^2 d\Omega^2_{(5)} \right), \\ H(y) = 1 + \left(\frac{R}{y} \right)^4, \\ F_{(5)} = d^4x \wedge dH^{-1} + \star\, d^4x \wedge dH^{-1}, \\ e^\Phi = g_s, \quad R^4 = 4\pi g_s N_c (\alpha')^2 \end{cases} \tag{3}$$

- Large N brane limit converts evectively the flat spacetime + N $D3$-branes into $AdS_5 \times S^5$ spacetime.

Cartoons for AdS/CFT correspondence



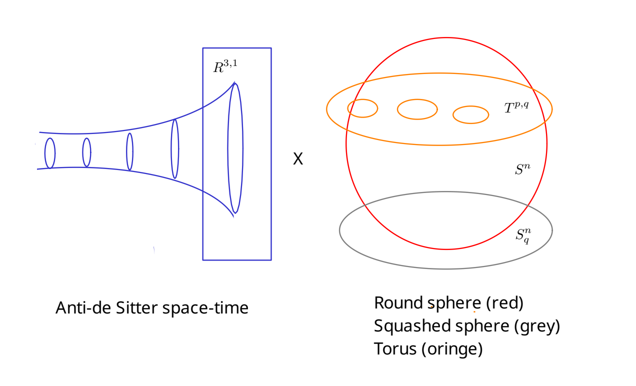


The left diagram showes how AdS/CFT correspondence is obtained. Large number of D3-branes bends the spacetime producing $AdS_5 \times S^5$.

The initial idea: (open) strings connect quark and antiquark. In holographic correspondence elementary and composite particles are understood in the sense of the picture above.

SUSY from background

- What is the role of the 5D Einstein space X_5?



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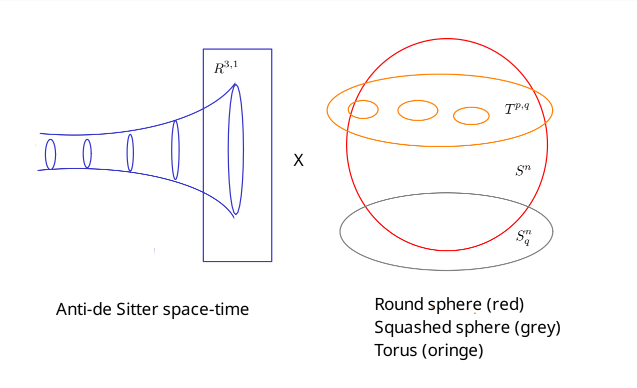

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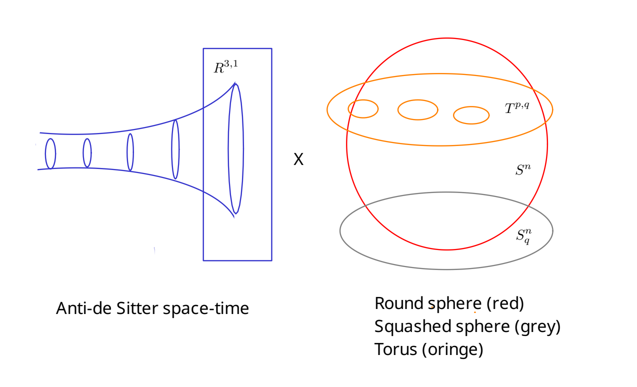

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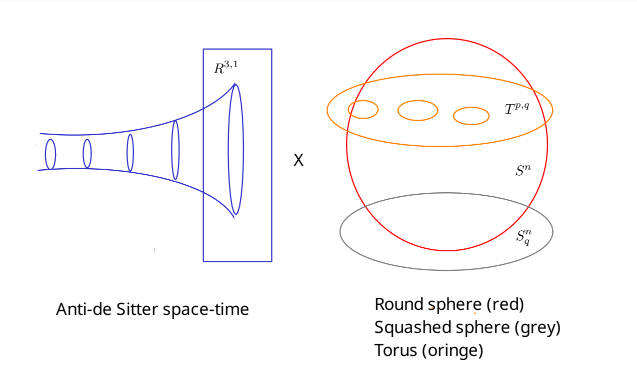

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- (Super-)Symmetry of $AdS_5 \times S^5 \to SU(2,2|4)$

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AdS/dS spaces: basics

- Start with n-dimensional sphere defined by the rotational $SO(n+1)$

$$S^n : \quad X_1^2 + X_2^2 + \cdots + X_{n+1}^2 = R^2. \tag{4}$$

- $\exists \; n(n+1)/2$ Killing vectors leaving the sphere invariant

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- Generalization: change the sign of the n+1-th coordinate ($X_{n+1} \equiv U$)

$$X_1^2 + X_2^2 + \cdots + X_n^2 - U^2 = -1. \tag{6}$$

  $n(n+1)/2$ vectors leave the hyperboloid invariant, but n independent

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- Let us change the sing in the metric of the embedding space, i.e. let us pass to Minkowski target space

$$ds^2 = dX_1^2 + \cdots + dX_n^2 - dU^2. \tag{8}$$

This metric is left invariant by all $n(n+1)/2$ vectors! The isometry group becomes $SO(n,1)$, i.e. the Lorentz group.

We define hyperbolic space $\mathbf{H}^n$ as the upper sheet of such a hyperboloid.

- Consider a one sheeted hyperboloid embedded in Minkowski space

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The space we obtain is a space with a Lorentzian metric of constant curvature and it is known under the name *de Sitter space*, $\mathbf{dS^n}$.

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- Consider the quadric

$$X_1^2 + \cdots + X_n^2 - X_{n-1}^2 - U^2 - V^2 = -1, \tag{10}$$

embedded in a flat n + 1 dimensional space with the metric

$$ds^2 = dX_1^2 + \cdots + dX_{n-1}^2 - dU^2 - dV^2. \tag{11}$$

The space we obtain is a space with a Lorentzian metric of constant curvature and it is known under the name *Anti-de Sitter space*, $\mathbf{AdS^n}$.

We define hyperbolic space $\mathbf{H}^n$ as the upper sheet of such a hyperboloid.

- Consider a one sheeted hyperboloid embedded in Minkowski space

$$X_1^2 + \cdots + X_n^2 - X_{n+1}^2 = 1. \tag{9}$$

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- Consider the quadric

$$X_1^2 + \cdots + X_n^2 - X_{n-1}^2 - U^2 - V^2 = -1, \tag{10}$$

embedded in a flat n + 1 dimensional space with the metric

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- $\mathbf{dS^n}$ space has space-like curves while $\mathbf{AdS^n}$ has time-like curves going around the hyperboloid.
- The topology of the two spaces
  - The topology of $\mathbf{AdS^n}$ is $\mathbf{R^{n-1}} \otimes \mathbf{S^1}$.
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- $d+1$-dimensional Euclidean and Minkowski AdS space as Lobachevski upper half plane $x^0 > 0$, $x^a = (x^0, \vec{x}); i = 1, \dots d$

$$ds^2 = \frac{1}{x_0^2}\left(dx^0 + d\vec{x}^2\right), \quad ds^2_{AdS} = L^2 \frac{dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu}{z^2}. \tag{12}$$

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- the vielbein and the corresponding spin connection are given by:

$$e_a^\mu = \frac{\delta_\mu^a}{x^0}; \quad \omega_i^{0j} = -\omega_i^{j0} = \frac{\delta_i^j}{x^0}; \quad a = 0, \dots d \tag{13}$$

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- Dirac operator

$$D_\nu=\partial_\nu+\frac{1}{2x^0}\gamma_{0\nu};\quad \Gamma^\mu D_\mu=x_0\gamma^0\partial_0+x_0\vec{\gamma}.\vec{\nabla}-\frac{d}{2}\gamma^0 \tag{16}$$

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$$\Psi_{\Sigma_\rho}[\phi_i] = \int_{\Phi_i|\Sigma_\rho=\phi_i} D\Phi_i e^{iS_{B,M}(\Phi_i)}, \tag{17}$$

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- The quantum Hamilton-Jacobi functional

$$S(\phi_i) = \Gamma(\Phi_i)|_{\frac{\delta\Gamma}{\delta\Phi_i}=0,\, \Phi_{i|\partial M}=\phi_i}. \tag{18}$$

- The fields on the boundary ϕ_i are characterized by their tensor structure and the conformal transformations of the boundary metric, namely

$$\gamma \to \rho^{-2}\gamma \quad \Longrightarrow \quad \phi_i \to \rho^{d-\Delta_i}\phi_i. \tag{19}$$

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- For any given CFT one can define the generating functional of connected correlation functions

$$Z_{CFT}[\phi_i] = \langle e^{\int_\Sigma \phi_i \mathcal{O}_i} \rangle. \tag{20}$$

The AdS/CFT correspondence states the equality of

$$\boxed{Z_{CFT}[\phi_i] = \Psi_{\Sigma_0}[\phi_i]}, \tag{21}$$

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- Changing the *radial slice* changes the induced metric on Σ_ρ! Thus, knowing Σ_ρ for all ρ (i.e. all possible γ) allows to reconstruct the semi-classical spacetime!
- On the other hand, assuming the correspondence, the variation of the boundary means moving the radial slice in the bulk!

- The correspondence states

$$Z_{CFT}[\phi_0] = \int D\mathcal{O} e^{iS_{\rm cft} + i\int d^dx\, \mathcal{O}(x)\phi_0(x)} = \int_{\Phi_{|=\phi_0}} D\Phi e^{iS_{\rm ads}} = \Psi_{\Sigma_\rho}[\phi_0]$$

- Practically

$$\langle \mathcal{O}_\Delta(x_1)\ldots\mathcal{O}_\Delta(x_n)\rangle_{\rm CFT,conn} = (-1)^n \frac{\delta^n \log \Psi_{\Sigma_\rho}[\phi_0]}{\delta\phi_0 \ldots \delta\phi_0}\Big|_{\phi_0=0}. \quad (22)$$

Example 1: N=4 SYM

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$\mathcal{N} = 4$ field content

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The $\mathcal{N} = 4$ vector multiplet consists of:
- the gauge field A_μ,
- four Weyl spinors λ_α^A, $A = 1, \ldots, 4$
- six real scalars X_m, $m = 4, \ldots, 9$ corresponding to the six transverse directions to the $D3$ branes.

It is convenient to represent the scalars as a self-dual antisymmetric tensor X^{AB} of the R-symmetry group $SU(4)_R \cong Spin(6)$,

$$(X^{AB})^\dagger = \bar{X}_{AB} \equiv \frac{1}{2}\epsilon_{ABCD}\, X^{CD}\,. \tag{23}$$

The explicit change of variables is

$$X^{AB} = \frac{1}{\sqrt{2}} \left(\begin{array}{cc|cc} 0 & X_8 + iX_9 & X_6 + iX_7 & X_4 + iX_5 \\ -X_8 - iX_9 & 0 & X_4 - iX_5 & -X_6 + iX_7 \\ \hline -X_6 - iX_7 & -X_4 + iX_5 & 0 & X_8 - iX_9 \\ -X_4 - iX_5 & X_6 - iX_7 & -X_8 + iX_9 & 0 \end{array} \right)\,.$$

$\Longrightarrow$ In components (in going from superspace to components, we redefine the coupling, $g_{\text{superspace}} = \sqrt{2} g_{\text{components}}$, to recover the usual normalization),

$$\mathcal{L}_{\mathcal{N}=4} = \mathrm{Tr}\Bigg[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - i \bar{\lambda}_A \bar{\sigma}^\mu D_\mu \lambda^A - \frac{1}{2} D^\mu \bar{X}_{AB} D_\mu X^{AB}$$

$$+ i\sqrt{2}\, g\, X^{AB}\, \bar{\lambda}_A \bar{\lambda}_B - i\sqrt{2}\, g\, \bar{X}_{AB} \lambda^A \lambda^B - \frac{g^2}{4} [X^{AB}, X^{CD}]\, [\bar{X}_{CD}, \bar{X}_{AB}] \Bigg], \tag{24}$$

where $A, B = 1, \ldots, 4$. The scalars X^{AB} are related to the three complex scalars ϕ^a as

$$X^{AB} = \left(\begin{array}{cc|cc} 0 & \phi^3 & \phi^2 & \phi^1 \\ -\phi^3 & 0 & \phi_1^* & -\phi_2^* \\ \hline -\phi^2 & -\phi_1^* & 0 & \phi_3^* \\ -\phi^1 & \phi_2^* & -\phi_3^* & 0 \end{array} \right) \tag{25}$$

and obey the self-duality constraint.

Example 2: N=2 SYM

The stack of branes: N_c $D3$ branes forming the geometry;. The background in the $N_c \to \infty$ limit approaches $AdS_5 \times S^5$. To introduce flavors we add N_F $D7$ branes according to the Table 3

Coordinates									
0	1	2	3	4	5	6	7	8	9
D3									
D7									
$x^{\mu,\nu,\dots}$				$y^{m,n,\dots}$				$z^{i,j,\dots}$	
				r					
				y					

Figure: *Embedding of N_F $D7$ branes in the background of N_c $D3$.*

The $AdS_5 \times S^5$ background is given by

$$ds^2 = H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)(d\vec{y}^2 + d\vec{z}^2),$$

$$H(r) = \frac{L^4}{r^4}, \quad r^2 = \vec{y}^2 + \vec{z}^2, \quad \vec{y}^2 = \sum_{m=4}^{7} y^m y^m, \quad \vec{z}^2 = (z^8)^2 + (z^9)^2$$

$$C^{(4)}_{0123} = H^{-1}, \quad e^\phi = e^{\phi_\infty} = g_s, \quad L^4 = 4\pi g_s N_c(\alpha')^2.$$

$\mathcal{N} = 2$ field content

Each $\mathcal{N} = 2$ flavor hyper multiplet consists of two Weyl spinors and two complex scalars,

$$\begin{array}{ccc} & \psi^i_\alpha & \\ q^i & & (\tilde{q}_i)^\dagger \\ & \left(\tilde{\psi}_{i\,\alpha}\right)^\dagger & \end{array} \tag{27}$$

Here $i = 1, \ldots, N_f$ is the flavor index. The scalars form an $SU(2)_R$ doublet,

$$Q^{\mathcal{I}} \equiv \begin{pmatrix} q \\ \tilde{q}^\dagger \end{pmatrix}, \quad \mathcal{I} = 1, 2\,. \tag{28}$$

The flavor hyper multiplets are minimally coupled to the $\mathcal{N} = 2$ vector multiplet that sits inside the $\mathcal{N} = 4$ vector multiplet. This coupling breaks the R-symmetry $SU(4)_R$ to $SU(2)_L \times SU(2)_R \times U(1)_R$, where $SU(2)_R \times U(1)_R$ is the R-symmetry of the resulting $\mathcal{N} = 2$ theory. There is a certain arbitrariness in the choice of embedding $SU(2)_L \times SU(2)_R \times U(1)_R \subset SU(4)_R \cong Spin(6)$.

$\Longrightarrow$ This corresponds to the choice of orientation of the whole stack of $D7$ branes in the 456789 directions (we need to pick an $\mathbb{R}^4 \subset \mathbb{R}^6$). For example if we choose the configuration of Figure, we identify $SU(2)_L \times SU(2)_R \cong SO(4)$ with rotations in the 4567 directions and $U(1)_R \cong SO(2)$ with a rotation on the 89 plane. A short calculation using our parametrization of the scalars (24) shows that this corresponds to the following natural embedding of $SU(2)_L \times SU(2)_R \times U(1)_R \subset SU(4)_R$:

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left(\begin{array}{c|c} SU_R(2) \times U(1)_R & \\ & \\ \hline & \\ & SU_L(2) \times U(1)_R^* \end{array} \right) . \tag{29}$$

Of course, any other choice would be equivalent, so long as it is performed simultaneously for all $D7$ branes.

$\Longrightarrow$ With the choice (29), the $\mathcal{N}=4$ vector multiplet splits into the $\mathcal{N}=2$ vector multiplet

$$\begin{array}{ccc} & A_\mu & \\ \lambda^1_\alpha & & \lambda^2_\alpha \\ & \frac{X_8+iX_9}{\sqrt{2}} & \end{array} \quad , \tag{30}$$

and the $\mathcal{N}=2$ hyper multiplet

$$\begin{array}{ccc} & \lambda^3_\alpha & \\ \frac{X_4+iX_5}{\sqrt{2}} & & \frac{X_6+iX_7}{\sqrt{2}} \\ & \lambda^4_\alpha & \end{array} \quad . \tag{31}$$

The two Weyl spinors in the vector multiplet form an $SU(2)_R$ doublet

$$\Lambda_{\mathcal{I}} \equiv \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad \mathcal{I}=1,2\,, \tag{32}$$

while the two spinors in the hyper multiplet form an $SU(2)_L$ doublet,

$$\hat{\Lambda}_{\hat{\mathcal{I}}} \equiv \begin{pmatrix} \lambda_3 \\ \lambda_4 \end{pmatrix}, \quad \hat{\mathcal{I}}=1,2. \tag{33}$$

We use $\mathcal{I}, \mathcal{J} \cdots = 1, 2$ for $SU(2)_R$ indices and $\hat{\mathcal{I}}, \hat{\mathcal{J}} \cdots = 1, 2$ for $SU(2)_L$ indices. To make the $SU(2)_L \times SU(2)_R$ quantum numbers of the scalars more transparent we also introduce the 2×2 complex matrix $\mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}}$, defined as the off-diagonal block of X^{AB},

$$\mathcal{X}^{\hat{\mathcal{I}}\mathcal{I}} = \left(\begin{array}{cc} X_6 + iX_7 & X_4 + iX_5 \\ X_4 - iX_5 & -X_6 + iX_7 \end{array} \right) . \tag{34}$$

Note that $\mathcal{X}^{\hat{\mathcal{I}}\mathcal{I}}$ obeys the reality condition

$$\left(\mathcal{X}^{\hat{\mathcal{I}}\mathcal{I}}\right)^* = -\mathcal{X}_{\hat{\mathcal{I}}\mathcal{I}} = -\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}\epsilon_{\mathcal{I}\mathcal{J}}\mathcal{X}^{\hat{\mathcal{J}}\mathcal{J}} . \tag{35}$$

We summarize in the following table the transformation properties of the fields:

	$SU(N)$	$SU(N_f)$	$SU(2)_L$	$SU(2)_R$	$U(1)_R$
A_μ	Adj	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0
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$\mathcal{X}^{\mathcal{I}\hat{\mathcal{I}}}$	Adj	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	0
$\Lambda_{\mathcal{I}}$	Adj	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	+1
$\hat{\Lambda}_{\hat{\mathcal{I}}}$	Adj	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	–1
$Q^{\hat{\mathcal{I}}}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{2}$	0
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In superfield language the Lagrangian reads

$$\mathcal{L}_{\mathcal{N}=4} = \mathrm{Tr}\left[\int d^4\theta\, e^{-gV}\,\bar{\Phi}_a\, e^{gV}\,\Phi^a + \int d^2\theta\, W^2 \right.$$
$$\left. + \left(\frac{i\,g}{3!}\int d^2\theta\,\epsilon_{abc}\,\Phi^a\left[\Phi^b,\,\Phi^c\right] + h.c.\right)\right]\,,$$

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$\odot$ If the above statement is true, the (quantum) gravity should be encoded in the boundary theory!
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▷ Is there something beyond the conjecture that could help in studying features of quantum gravity, or what is underlying this duality ?

Next lectures will demonstrate how to approach these problems and will give simple examples of duality with focus on the field theory side.